

# Graph Neural Networks for Recommendation: Reproducibility, Graph Topology, and Node Representation

**Part 2.** Graph topology

 30 minutes

TUTORIALS



The 2nd Learning on Graphs Conference (LoG 2023)

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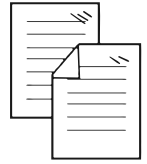
# USEFUL RESOURCES

The content of the following slides is taken from:

- Daniele Malitesta, Claudio Pomo, Vito Walter Anelli, Alberto Carlo Maria Mancino, Eugenio Di Sciascio, Tommaso Di Noia:  
**A Topology-aware Analysis of Graph Collaborative Filtering.**  
CoRR abs/2308.10778 (2023)



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# 01

## MOTIVATIONS

# WHY GRAPH CF WORKS WELL?

## Algorithmically

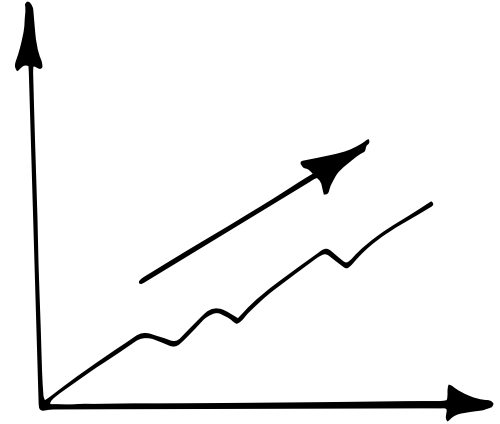
- **Established** approaches such as **LightGCN** and **DGCF** adapt the **GCN layer** to suit the **CF rationale**.
- **Recent** graph-based RSs including **UltraGCN** and **SVD-GCN** go **beyond** the concept of **message-passing**.

## Topologically

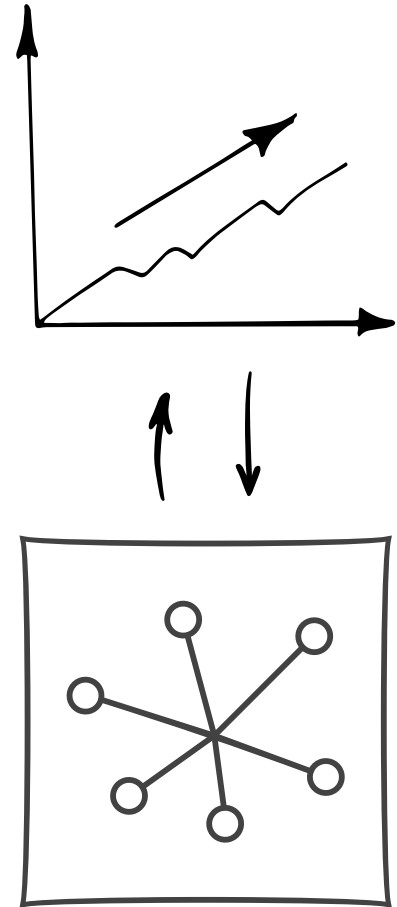
- The **machine learning** literature acknowledges that **graph topology** plays a **crucial role** in **GNNs** [Wei et al., Castellana and Errica].
- For instance, revisit **network topology** to **enhance** the **learning ability** of **GCNs** [Shi et al.].

Classical dataset characteristics such as **dataset sparsity** may impact the **performance** of **recommendation** models in diverse scenarios and tasks.

[Adomavicius and Zhang, Deldjoo et al.]



The **topological** nature of the **user-item data** and the new **graph CF wave** make it imperative to **unveil** the **dependencies** between **topological properties** and **recommendation performance**.



# OUR CONTRIBUTIONS

## 1) First analysis in the literature

that studies the **impact** of **classical** and **topological** dataset characteristics on the performance of **SOTA graph CF**.

## 3) Build an explanatory model

to calculate the **linear dependences** among **characteristics** and recommendation **metrics**.

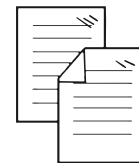
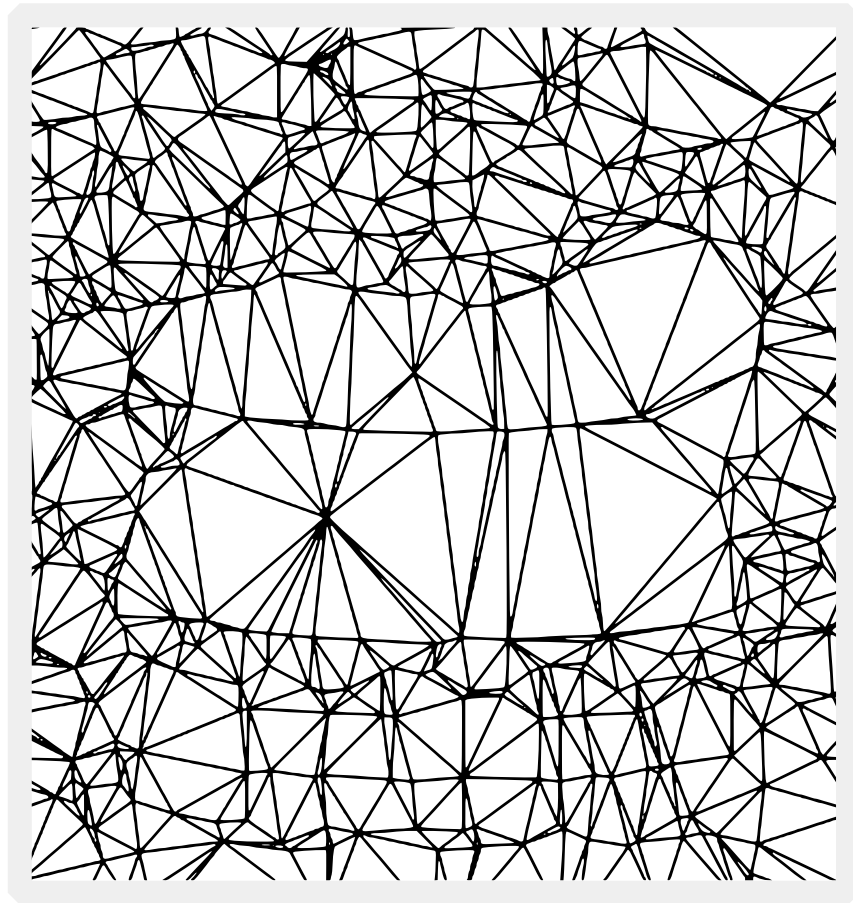
## 2) Select 4 graph CF approaches

which are **recent** and **across-the-board** in the literature.

## 4) Validate the explanations

on their **statistical significance**, with **varying** settings of **graph samplings**.





# 02

## TOPOLOGICAL CHARACTERISTICS

# NODE DEGREE EXTENSION (1/3)

## DEFINITION

Let  $N_u^{(l)}$  and  $N_i^{(l)}$  be the sets of neighborhood nodes for user  $u$  and item  $i$  at  $l$  distance hops. The extended definition of node degrees for  $u$  and  $i$  is:

$$\sigma_u = |N_u^{(1)}|, \quad \sigma_i = |N_i^{(1)}|.$$

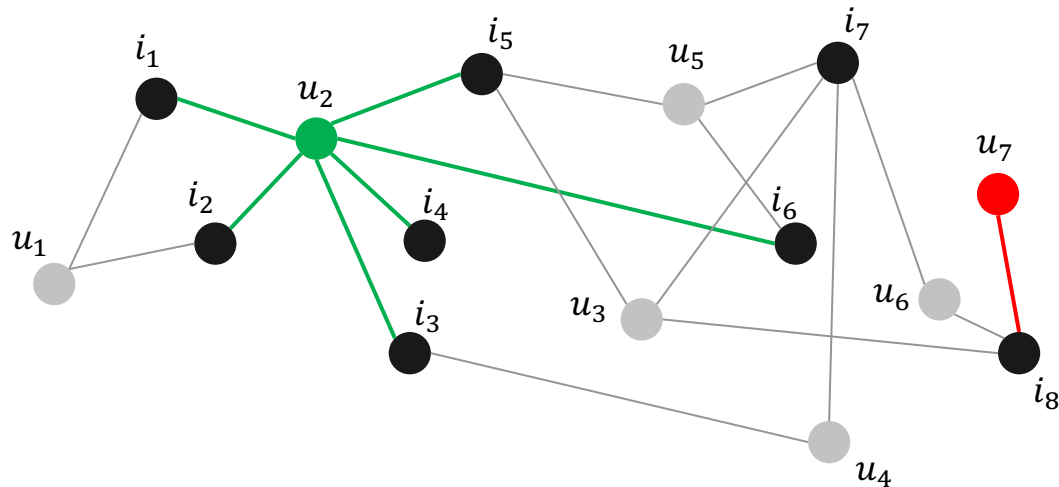
The average user and item node degrees on the whole user and item sets ( $U$  and  $I$ ):

$$\sigma_U = \frac{1}{U} \sum_{u \in U} |N_u^{(1)}|, \quad \sigma_I = \frac{1}{I} \sum_{i \in I} |N_i^{(1)}|.$$

# NODE DEGREE EXTENSION (2/3)

## RECSYS RE-INTERPRETATION

The **node degree** in the **user-item graph** stands for the **number of items (users) interacted by a user (item)**. This is related to the **cold-start issue** in recommendation, where **cold users** denote **low activity** on the platform, while **cold items** are **niche products**.



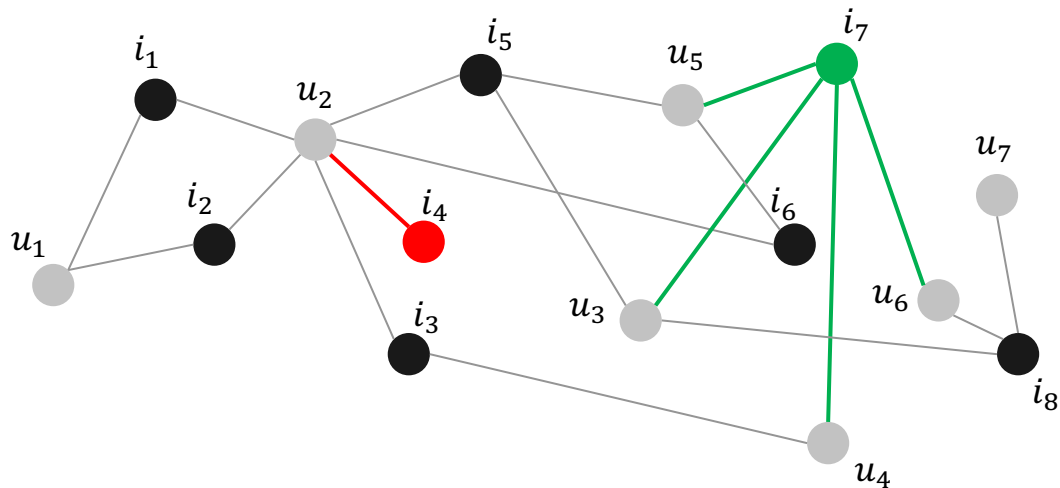
$$\sigma_{u_2} = 5 \text{ (active user)}$$

$$\sigma_{u_7} = 1 \text{ (non-active user)}$$

# NODE DEGREE EXTENSION (3/3)

## RECSYS RE-INTERPRETATION

The **node degree** in the **user-item graph** stands for the **number of items (users)** interacted by a **user (item)**. This is related to the **cold-start issue** in recommendation, where **cold users** denote **low activity** on the platform, while **cold items** are **niche products**.



$$\sigma_{i_7} = 4 \text{ (popular item)}$$

$$\sigma_{i_4} = 1 \text{ (niche item)}$$

# CLUSTERING COEFFICIENT (1/5)

## DEFINITION

Let  $v$  and  $w$  be two nodes from the same partition (e.g., user nodes). Their similarity is the intersection over union of their neighborhoods. By evaluating the metric node-wise:

$$\gamma_v = \frac{\sum_{w \in N_v^{(2)}} \gamma_{v,w}}{|N_v^{(2)}|} \quad \text{with } \gamma_{v,w} = \frac{|N_v^{(1)} \cap N_w^{(1)}|}{|N_v^{(1)} \cup N_w^{(1)}|},$$

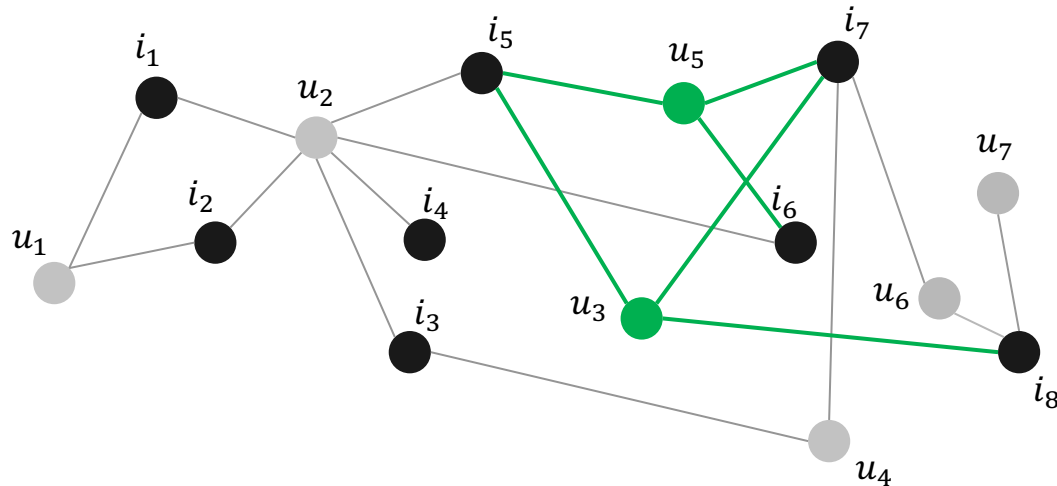
where  $N_v^{(2)}$  is the second-order neighborhood set of  $v$ . The average clustering coefficient [Latapy et al.] on  $U$  and  $I$  is:

$$\gamma_U = \frac{1}{U} \sum_{u \in U} \gamma_u. \quad \gamma_I = \frac{1}{I} \sum_{i \in I} \gamma_i.$$

# CLUSTERING COEFFICIENT (2/5)

## RECSYS RE-INTERPRETATION

High values of the **clustering coefficient** indicate that there exists a **substantial number** of **co-occurrences** among **nodes** of the **same partition**. For instance, **user-wise**, the value **increases** if **several users share** most of their **interacted items**. This intuition **aligns** with the rationale of **collaborative filtering**: two users are likely to **share similar preferences** if they interact with the **same items**.

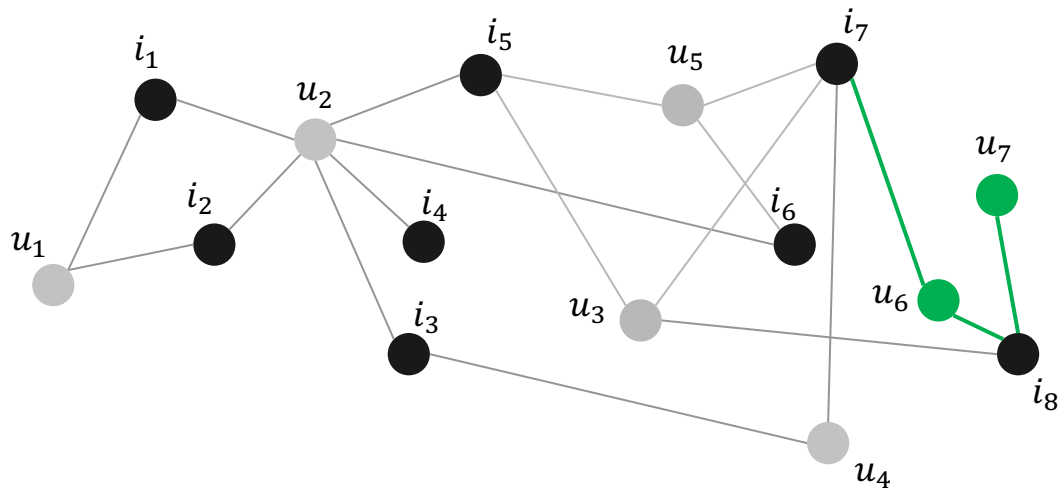


$$\gamma_{u_3, u_5} = \frac{|\{i_5, i_7\} \cap \{i_5, i_6, i_7\}|}{|\{i_5, i_6, i_7\}|} = \frac{2}{3} = 0.667$$

# CLUSTERING COEFFICIENT (3/5)

## RECSYS RE-INTERPRETATION

High values of the **clustering coefficient** indicate that there exists a **substantial number** of **co-occurrences** among **nodes** of the **same partition**. For instance, **user-wise**, the value **increases** if **several users share** most of their **interacted items**. This intuition **aligns** with the rationale of **collaborative filtering**: two users are likely to **share similar preferences** if they interact with the **same items**.

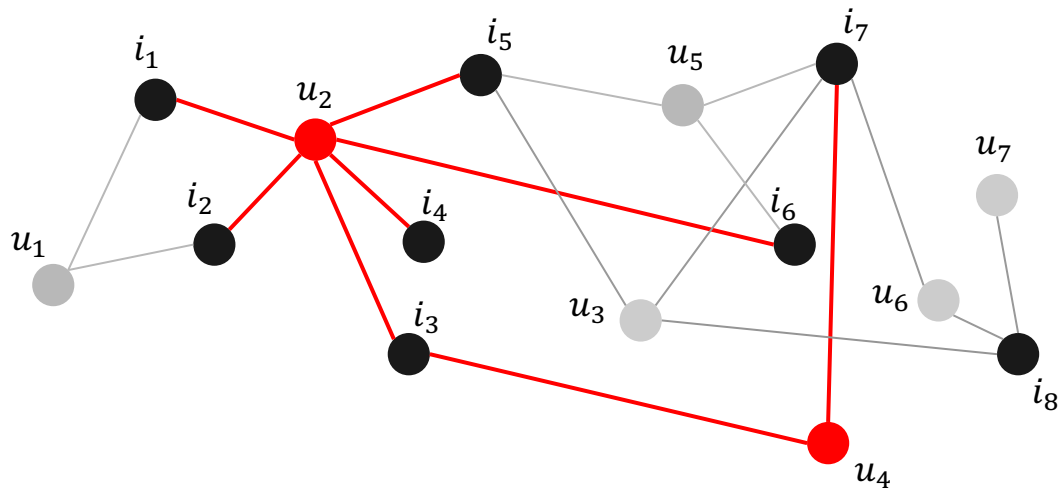


$$\gamma_{u_6, u_7} = \frac{|\{i_7, i_8\} \cap \{i_8\}|}{|\{i_7, i_8\}|} = \frac{1}{2} = 0.500$$

# CLUSTERING COEFFICIENT (4/5)

## RECSYS RE-INTERPRETATION

High values of the **clustering coefficient** indicate that there exists a **substantial number of co-occurrences** among **nodes of the same partition**. For instance, **user-wise**, the value **increases** if **several users share** most of their **interacted items**. This intuition **aligns** with the rationale of **collaborative filtering**: two users are likely to **share similar preferences** if they interact with the **same items**.



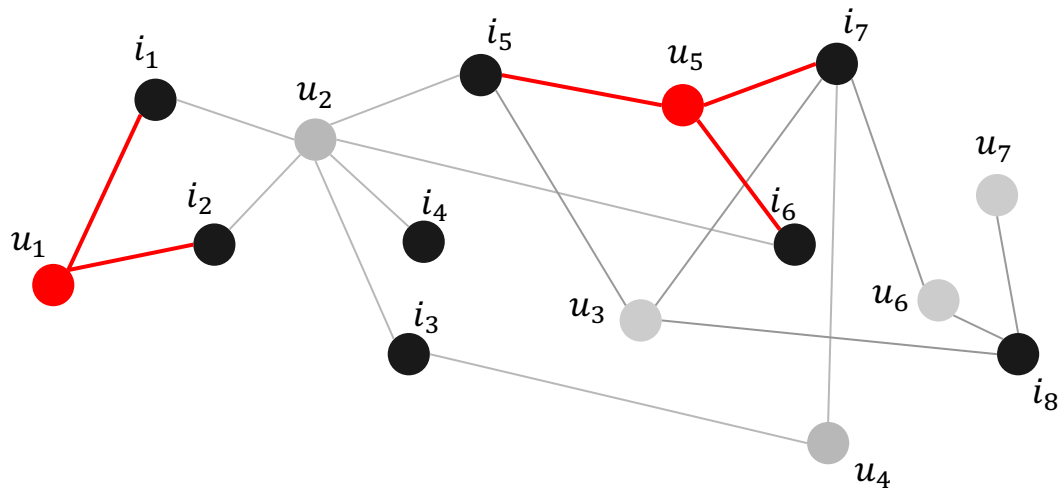
$$\gamma_{u_2, u_4} = \frac{|\{i_1, i_2, i_3, i_4, i_5\} \cap \{i_3, i_7\}|}{|\{i_1, i_2, i_3, i_4, i_5, i_7\}|} = \frac{1}{6} = 0.167$$



# CLUSTERING COEFFICIENT (5/5)

## RECSYS RE-INTERPRETATION

High values of the **clustering coefficient** indicate that there exists a **substantial number** of **co-occurrences** among **nodes** of the **same partition**. For instance, **user-wise**, the value **increases** if **several users share** most of their **interacted items**. This intuition **aligns** with the rationale of **collaborative filtering**: two users are likely to **share similar preferences** if they interact with the **same items**.



$$\gamma_{u_1, u_5} = \frac{|\{i_1, i_2\} \cap \{i_5, i_6, i_7\}|}{|\{i_1, i_2, i_5, i_6, i_7\}|} = \frac{0}{6} = 0.000$$

# DEGREE ASSORTATIVITY (1/3)

## DEFINITION

Let  $D = \{d_1, d_2, \dots\}$  be the set of **unique node degrees** in the graph, and let  $e_{d_h, d_k}$  be the **fraction of edges** connecting **nodes with degrees**  $d_h$  and  $d_k$ . Then, let  $q_{d_h}$  be the **probability distribution** to choose a node with degree  $d_h$  **after** having **selected a node** with the **same degree** (i.e., the **excess degree** distribution). The **degree assortativity** coefficient [M. E. J. Newman] is:

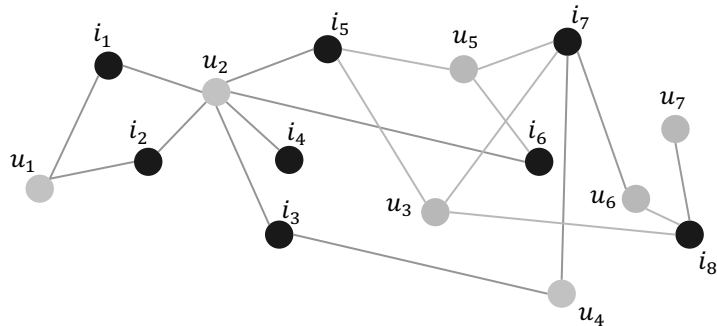
$$\rho = \frac{\sum_{d_h d_k} d_h d_k (e_{d_h, d_k} - q_{d_h} q_{d_k})}{std_q^2},$$

where  $std_q$  is the standard deviation of the distribution  $q$ .

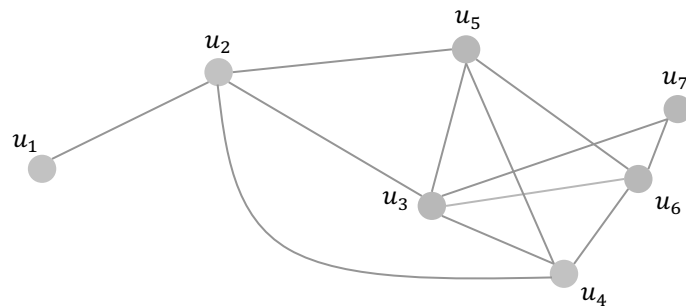
# DEGREE ASSORTATIVITY (2/3)

## RECSYS RE-INTERPRETATION

The **degree assortativity** calculated user- and item-wise is a **proxy** to represent the **tendency of users** with the **same activity** level on the platform and **items** with the **same popularity** to gather, respectively. To **visualize** the degree assortativity, we need the projected **user-user** and **item-item** graphs.



user-item graph

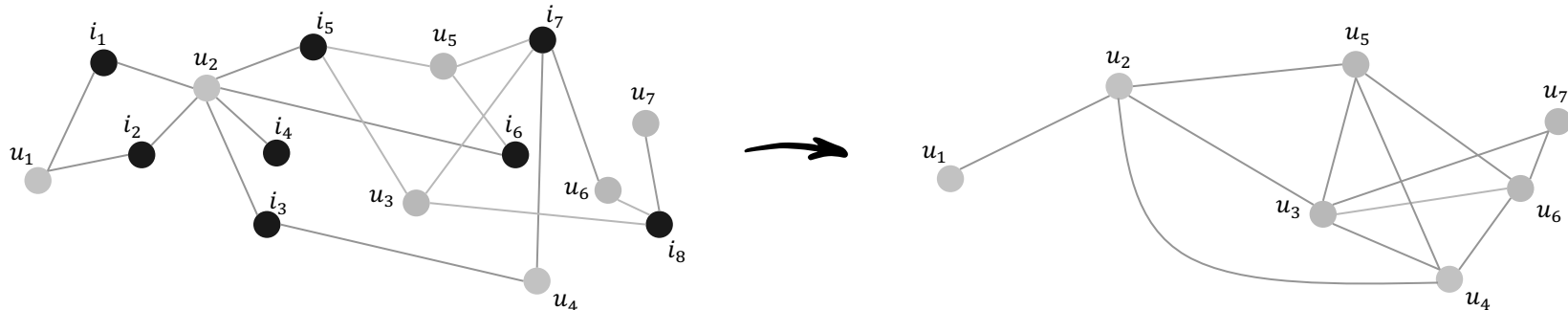


user-user projected graph

# DEGREE ASSORTATIVITY (3/3)

## RECSYS RE-INTERPRETATION

The **degree assortativity** calculated user- and item-wise is a **proxy** to represent the **tendency of users** with the **same activity** level on the platform and **items** with the **same popularity** to gather, respectively. To **visualize** the degree assortativity, we need the projected **user-user** and **item-item** graphs.



$\rho = -0.191$   
(high degree dis-assortativity)

# CLASSICAL DATASET CHARACTERISTICS

## Space size

Estimates the number of all possible interactions:

$$\zeta = \sqrt{UI}.$$

## Density

Measures the ratio of actual user-item interactions to all possible interactions:

$$\delta = \frac{E}{UI}$$

## Shape

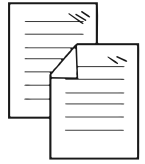
Defines the ratio between the number of users and items:

$$\pi = \frac{U}{I}$$

## Gini coefficient

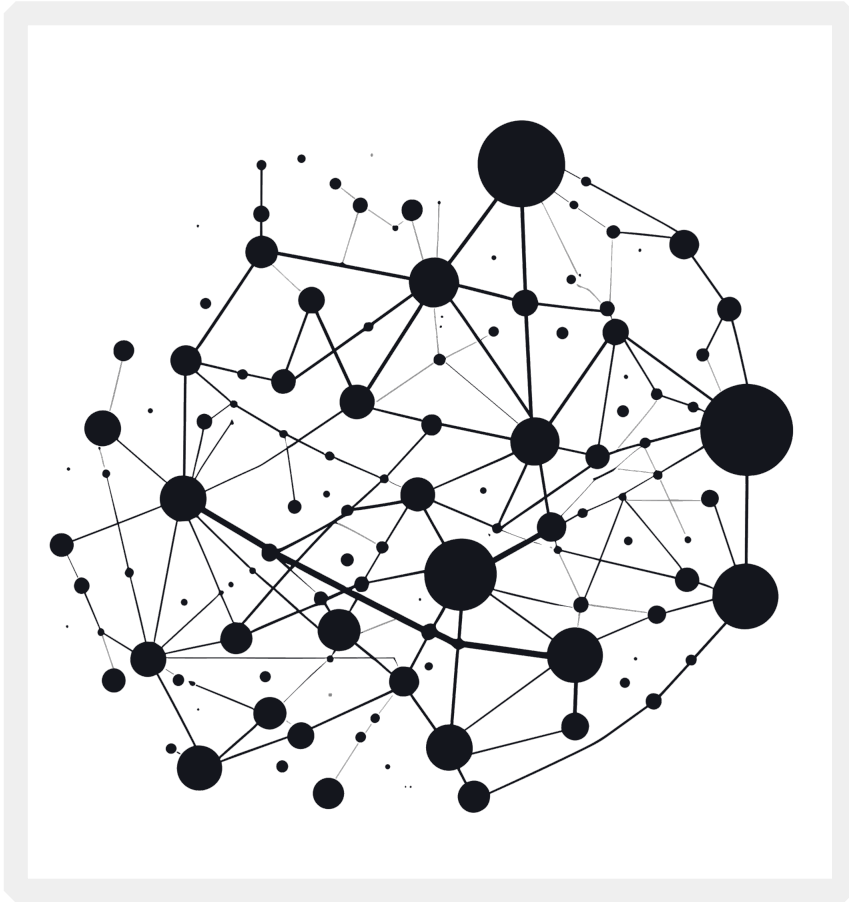
Calculates the interactions' concentration for users (items):

$$\kappa_U = \frac{\sum_{u=1}^{U-1} \sum_{v=u+1}^U \text{abs}(\sigma_u - \sigma_v)}{U \sum_{u=1}^U \sigma_u}$$



# 03

## TOPOLOGY IN GRAPH CF



# SELECTED GRAPH CF APPROACHES

## LightGCN (SIGIR 2020)

Node degree used to **normalize** the adjacency matrix in the message-passing.

## UltraGCN (CIKM 2021)

Node degree used for **normalization** in the **infinite layer message-passing**. The model also learns from the **item-projected graph**.

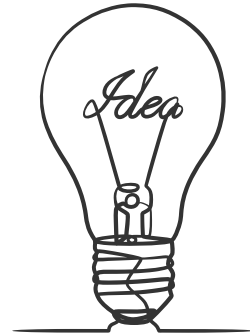
## DGCF (SIGIR 2020)

Node degree used to **normalize** the adjacency matrix in the message-passing.

## SVD-GCN (CIKM 2022)

Node embeddings involve the **largest singular values** of the **normalized user-item interaction matrix**, whose **maximum value** is related to the **maximum node degree** of the user-item graph. The model learns from **user- and item-projected graphs**.

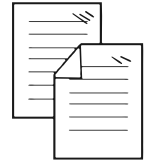
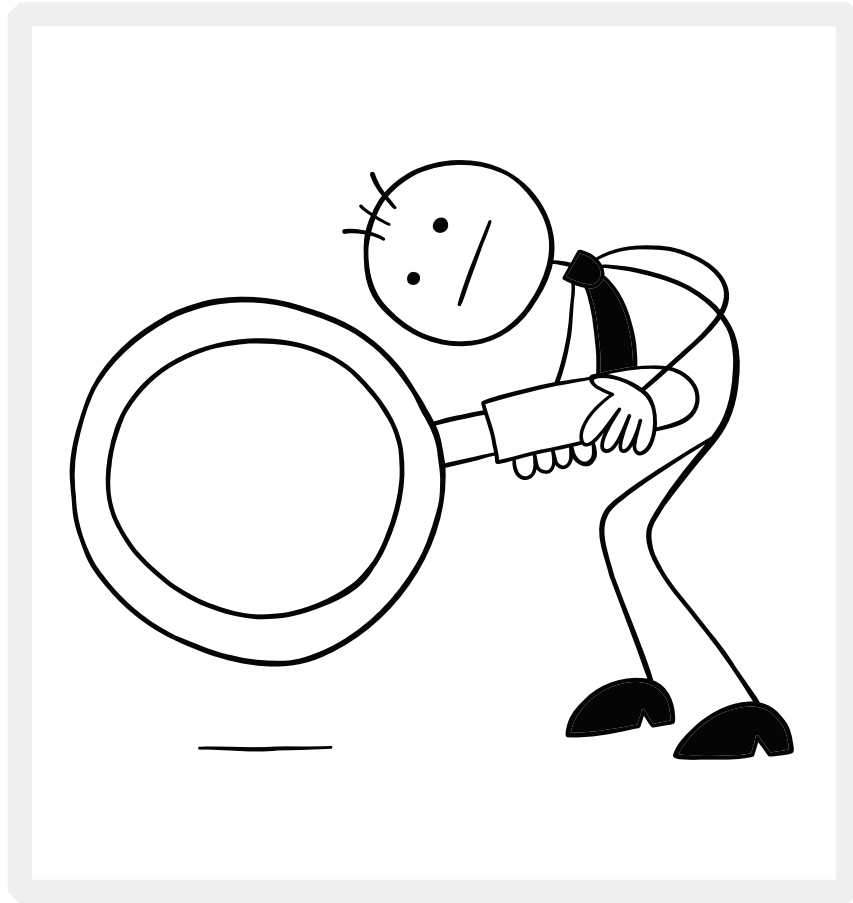
The **selected graph CF** approaches **explicitly** utilize the **node degree** in the representation learning. However, **clustering coefficient** and **degree assortativity** do **not** have an **evident representation** in the formulations.





- Which **topological aspects** graph-based models can **(un)intentionally capture**?
- Are (topological) dataset **characteristics influencing** the recommendation **performance** of graph CF models?



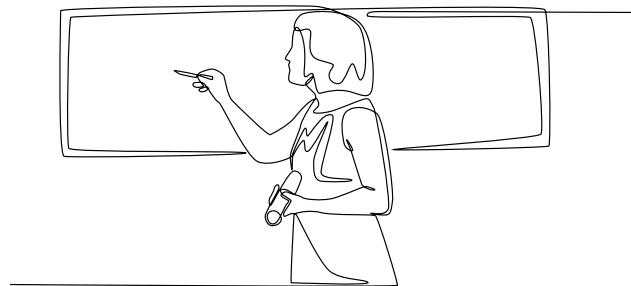


04

**PROPOSED  
ANALYSIS**

Our **goal** is to understand whether there **exist dependencies** among (topological) dataset **characteristics** and the **performance** of **graph-based** recommendation **models**.

To this aim, we decide to **build** and **fit** an **explanatory framework**.




# EXPLANATORY FRAMEWORK: STEPS

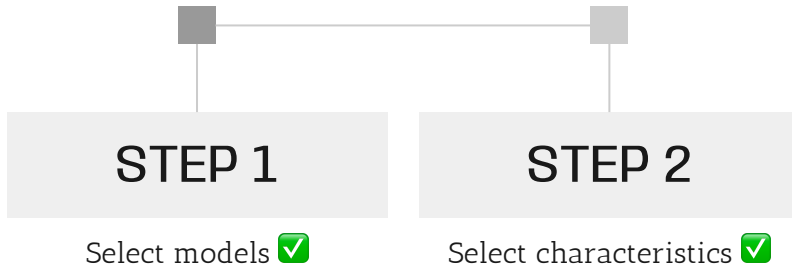
# EXPLANATORY FRAMEWORK: STEPS



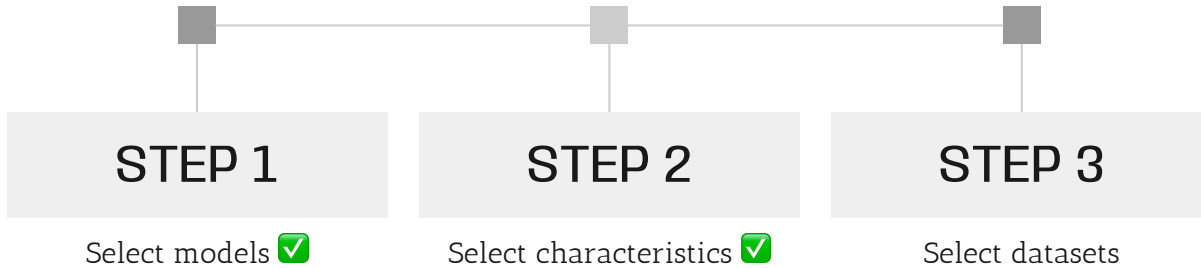
## STEP 1

Select models 

# EXPLANATORY FRAMEWORK: STEPS



# EXPLANATORY FRAMEWORK: STEPS

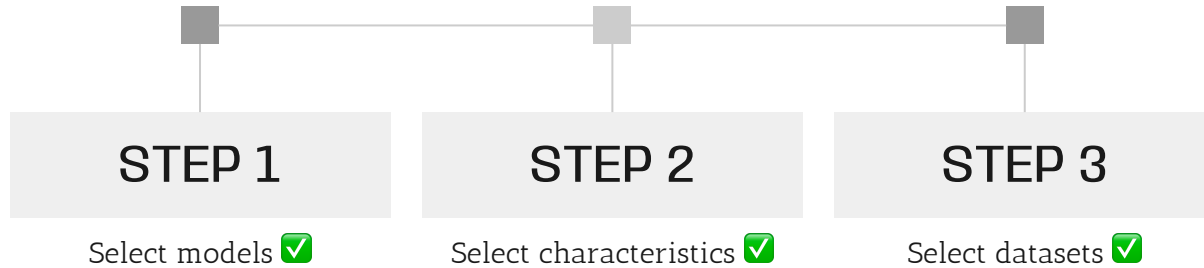


# DATASETS

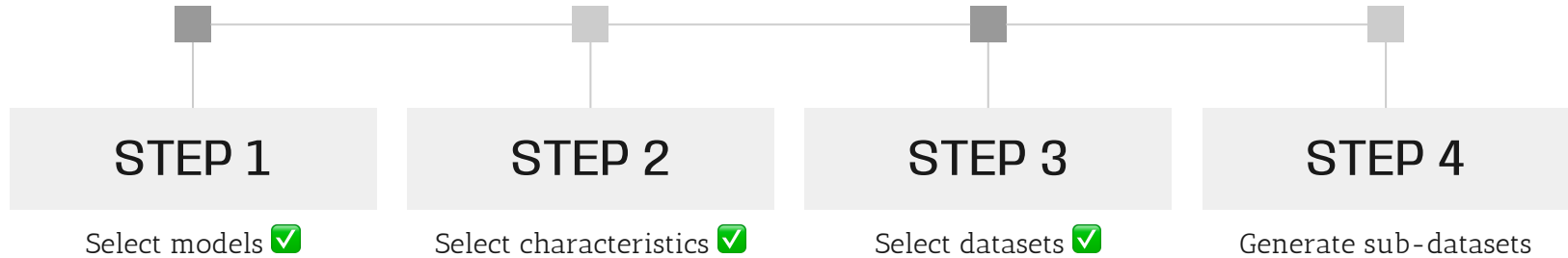
	# Users	# Items	# Interactions
Yelp-2018	25,677	25,815	696,865
Amazon -Book	70,679	24,915	846,434
Gowalla	29,858	40,981	1,027,370



# EXPLANATORY FRAMEWORK: STEPS



# EXPLANATORY FRAMEWORK: STEPS



# SUB-DATASETS GENERATION

---

**Algorithm 2:** Sub-dataset generation.

---

**Input:** Bipartite user-item graph  $\mathcal{G}$ , number of samples  $M$ .

**Output:**  $M$  sampled graphs.

$m \leftarrow 1$

$\mathcal{M} = \{\}$

**while**  $m \leq M$  **do**

$\mu \leftarrow \text{uniform}([0.7, 0.9])$

$\text{sampling} \leftarrow \text{uniform}(\{\text{nodeDropout}, \text{edgeDropout}\})$

$\mathcal{M} \leftarrow \mathcal{M} \cup \text{sample}(\mathcal{G}, \mu, \text{sampling})$

$m \leftarrow m + 1$

**end**

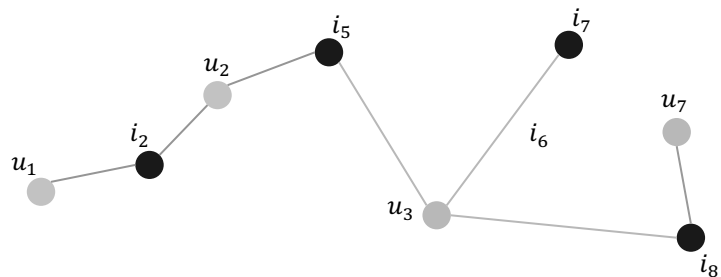
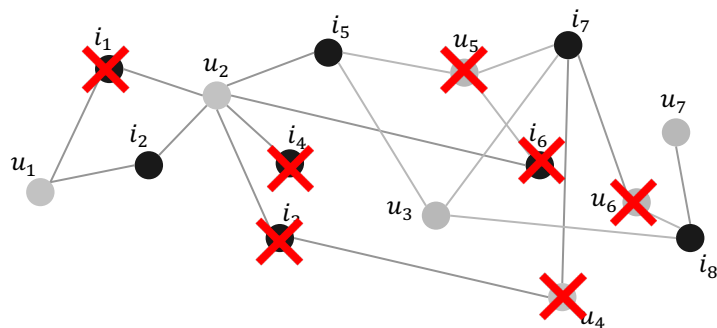
Return  $\mathcal{M}$ .

---





# NODE DROPOUT



users

1	1	0	0	0	0	0	0
1	1	1	1	1	1	0	0
0	0	0	0	1	0	1	1
0	0	1	0	0	0	1	0
0	0	0	0	1	1	1	0
0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	1

items

users

1	0	0	0
1	1	0	0
0	1	1	1
0	0	0	1

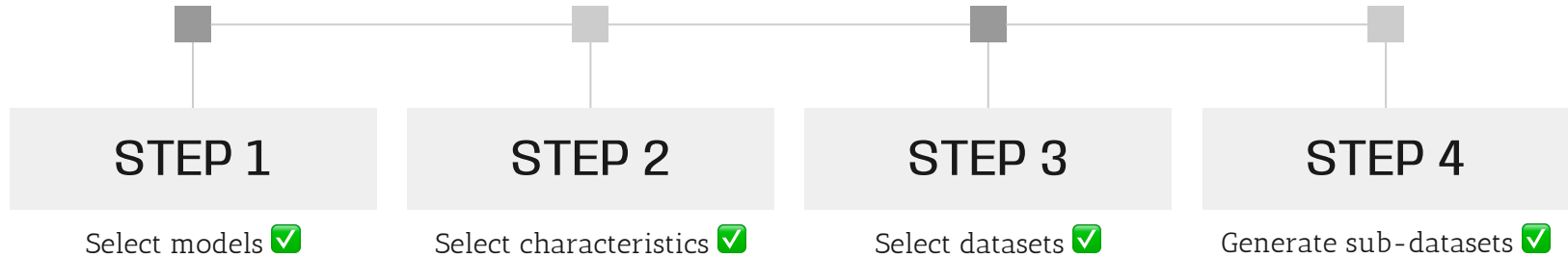
items



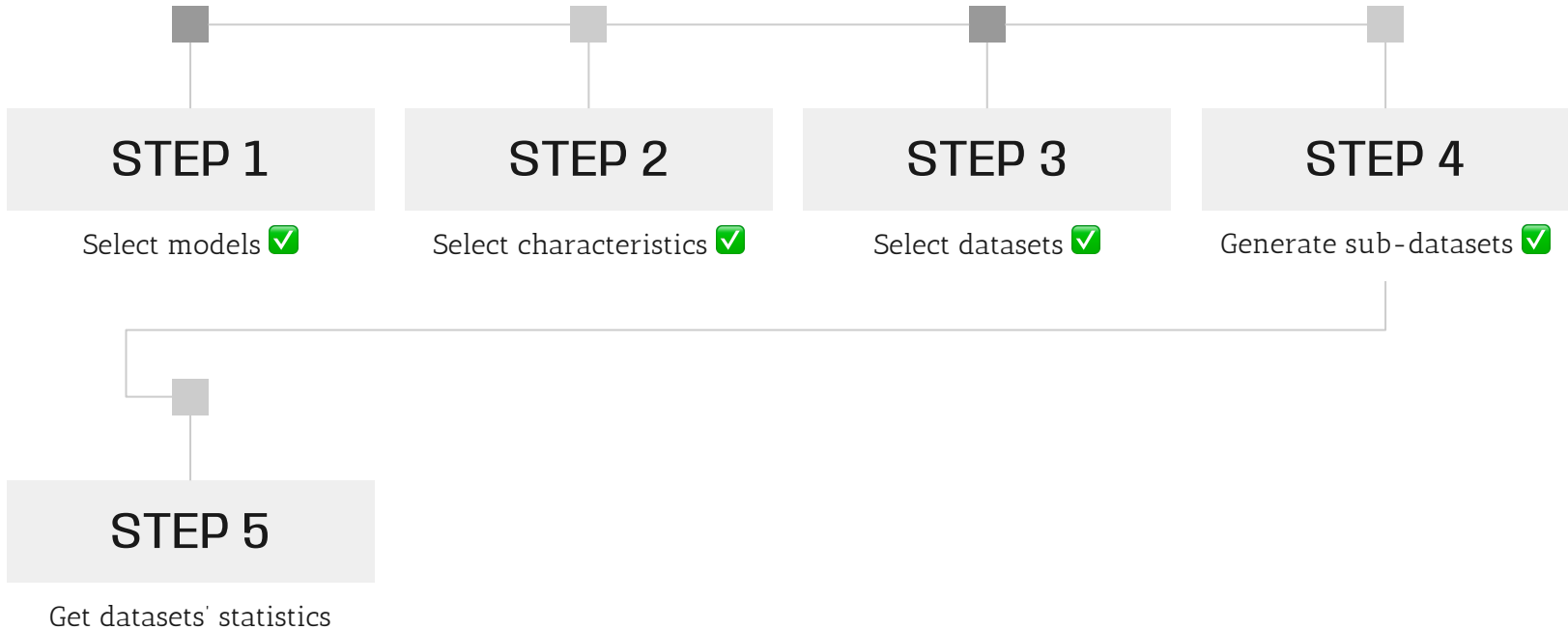




# EXPLANATORY FRAMEWORK: STEPS



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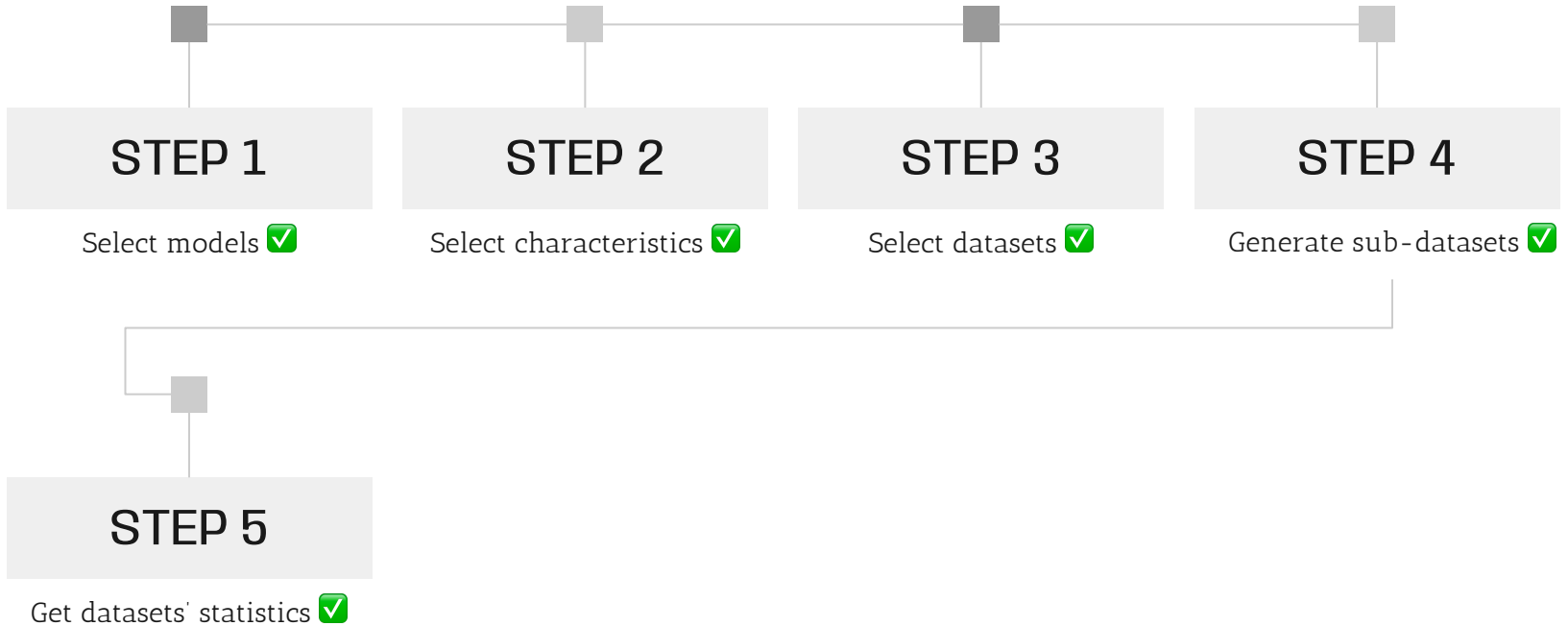


# STATISTICS CALCULATION (1/2)

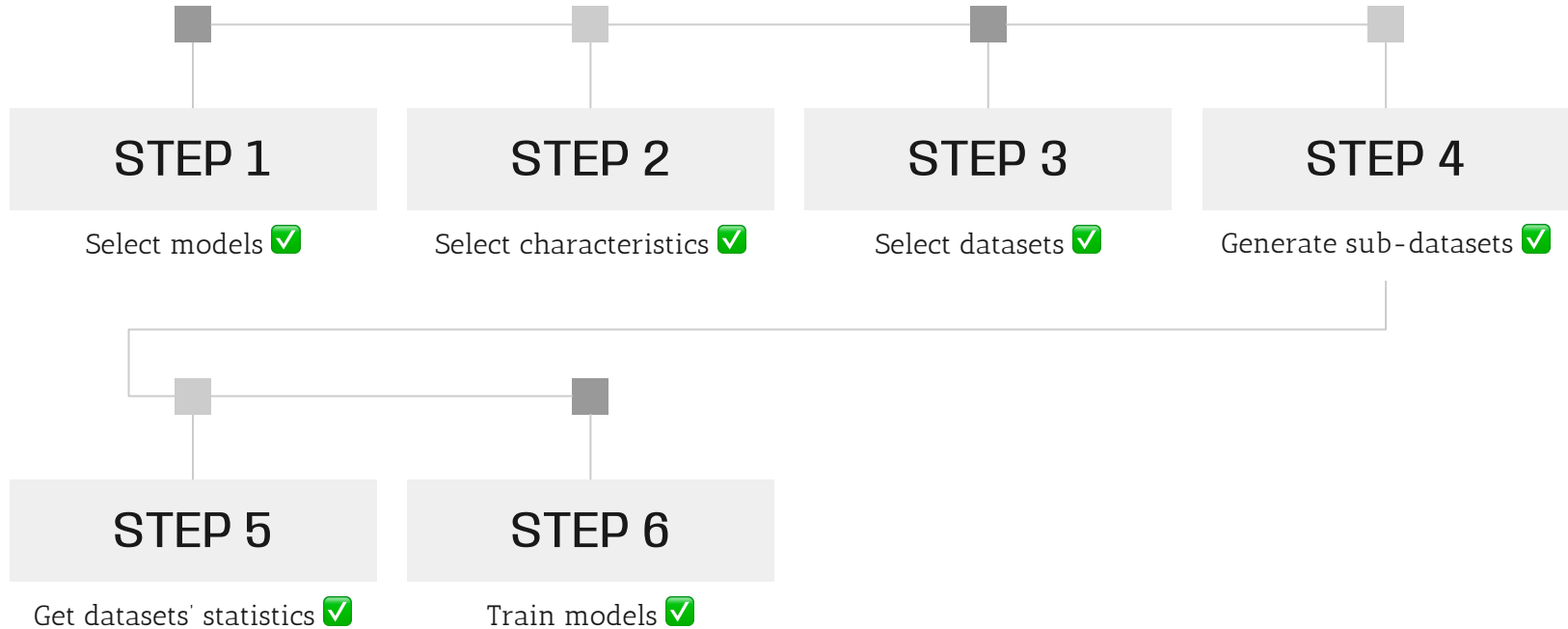
Type	Characteristics	Symbol	Log10	Shorthand
<i>Classical</i>	Space size	$\zeta$	✓	<i>SpaceSize<sub>log</sub></i>
	Shape	$\pi$	✓	<i>Shape<sub>log</sub></i>
	Density	$\delta$	✓	<i>Density<sub>log</sub></i>
	Gini user	$\kappa_U$		<i>Gini-U</i>
	Gini item	$\kappa_I$		<i>Gini-I</i>
<i>Topological</i>	Average degree user	$\sigma_U$	✓	<i>AvgDegree-U<sub>log</sub></i>
	Average degree item	$\sigma_I$	✓	<i>AvgDegree-I<sub>log</sub></i>
	Average clustering coefficient user	$\gamma_U$	✓	<i>AvgClustC-U<sub>log</sub></i>
	Average clustering coefficient item	$\gamma_I$	✓	<i>AvgClustC-I<sub>log</sub></i>
	Degree assortativity user	$\rho_U$		<i>Assort-U</i>
Degree assortativity item	$\rho_I$		<i>Assort-I</i>	



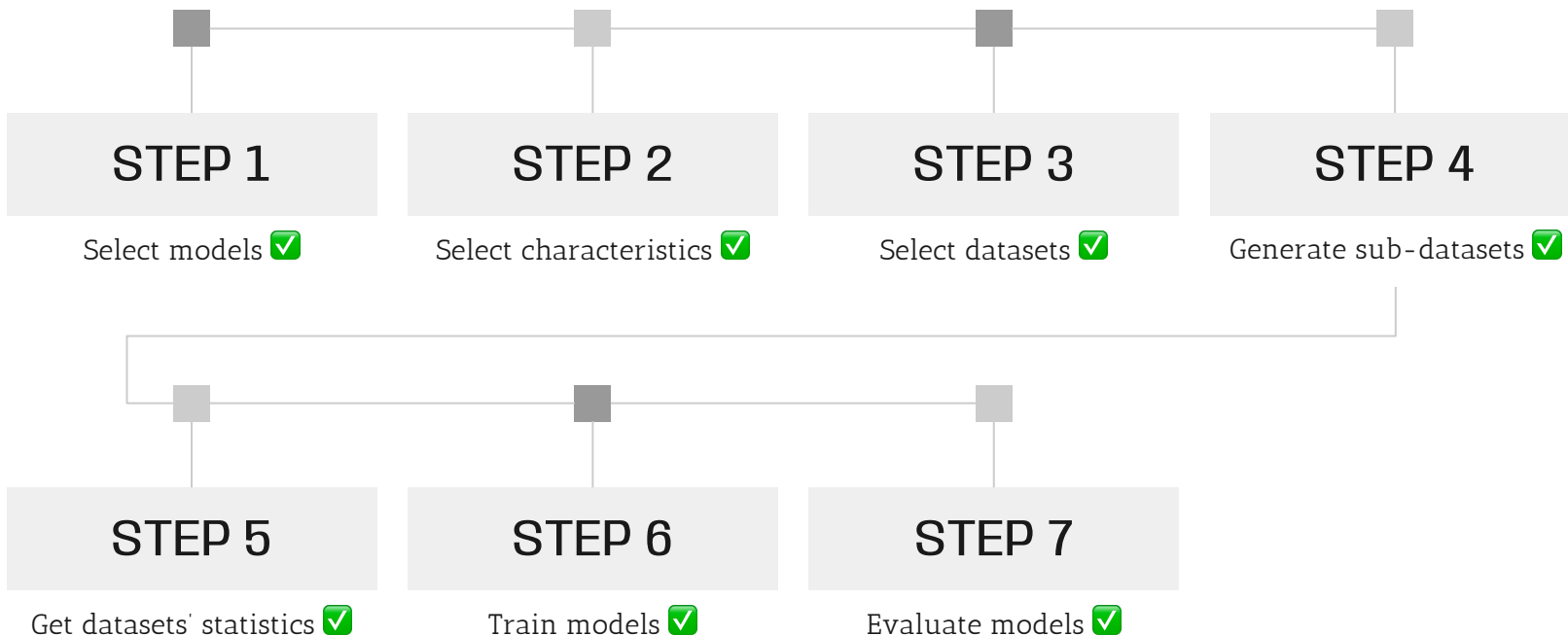
# EXPLANATORY FRAMEWORK: STEPS



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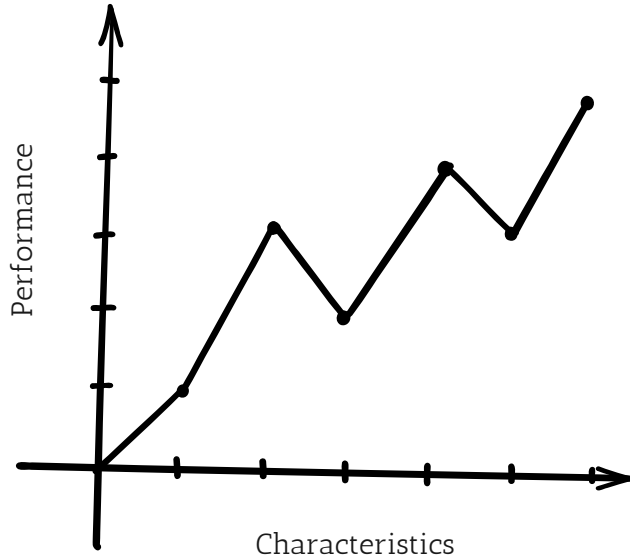


# EXPLANATORY FRAMEWORK: STEPS





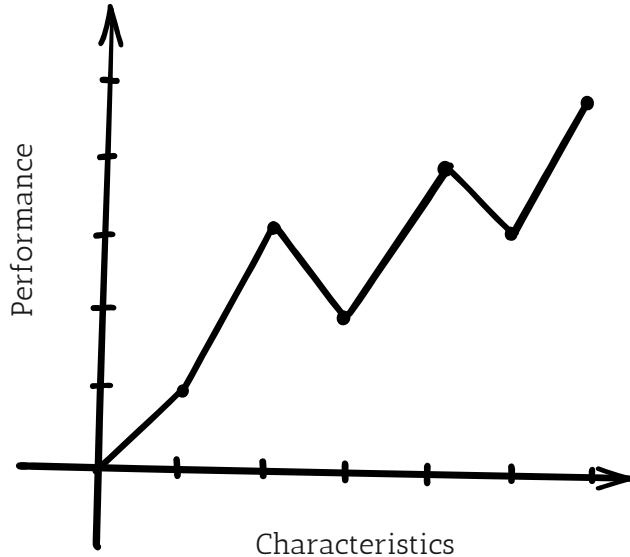
# GENERATE (LINEAR) EXPLANATIONS



We aim to fit the following multivariate linear regression model:

$$y = \epsilon + \theta_0 + \theta_c X_c.$$

# GENERATE (LINEAR) EXPLANATIONS

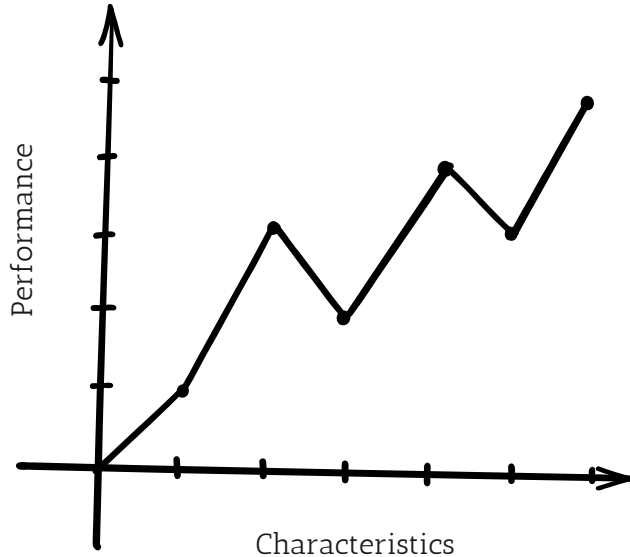


We aim to fit the following multivariate linear regression model:

$$\mathbf{y} = \epsilon + \theta_0 + \theta_c \mathbf{X}_c.$$

Predicted  
performance

# GENERATE (LINEAR) EXPLANATIONS



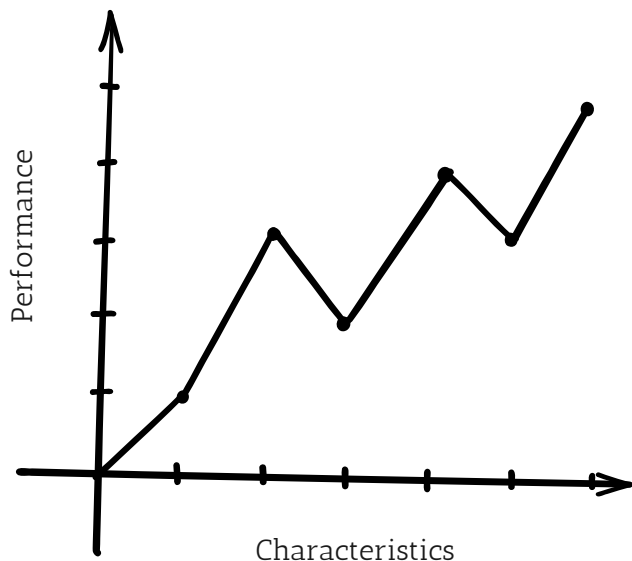
We aim to fit the following multivariate linear regression model:

$$\mathbf{y} = \epsilon + \theta_0 + \theta_c \mathbf{X}_c.$$

Predicted performance

Prediction error

# GENERATE (LINEAR) EXPLANATIONS



We aim to fit the following multivariate linear regression model:

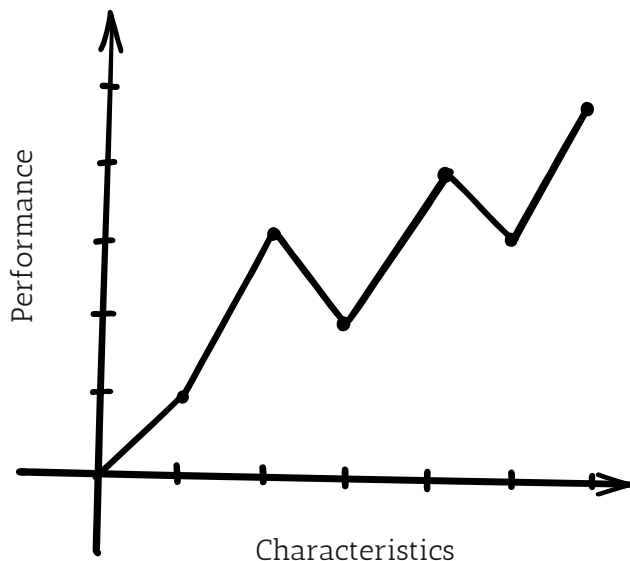
$$\mathbf{y} = \epsilon + \theta_0 + \theta_c \mathbf{X}_c.$$

Predicted performance

Prediction error

Global bias

# GENERATE (LINEAR) EXPLANATIONS



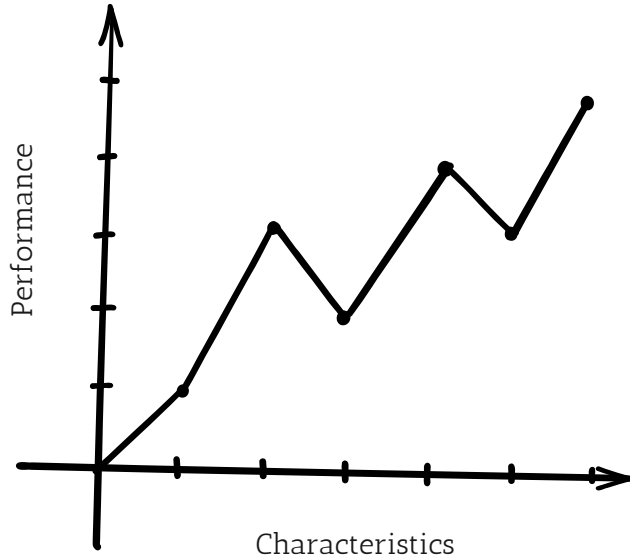
We aim to fit the following multivariate linear regression model:

$$y = \epsilon + \theta_0 + \theta_c X_c.$$

Diagram illustrating the components of the multivariate linear regression model equation  $y = \epsilon + \theta_0 + \theta_c X_c$ :

- $y$ : Predicted performance
- $\epsilon$ : Prediction error
- $\theta_0$ : Global bias
- $\theta_c X_c$ : Weight of characteristic  $c$

# GENERATE (LINEAR) EXPLANATIONS



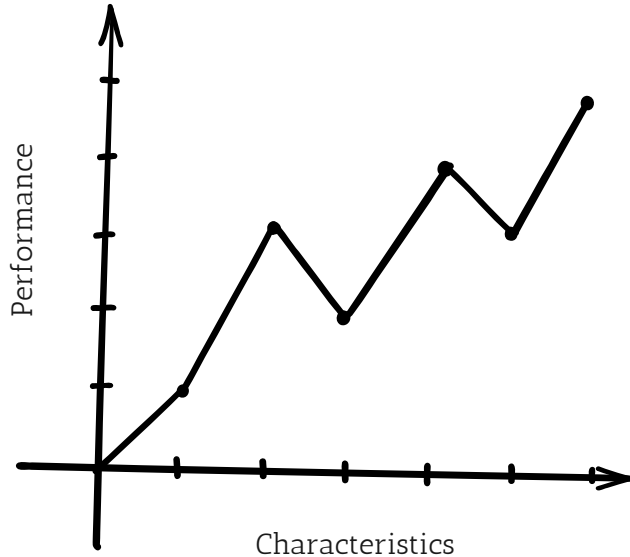
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Diagram illustrating the components of the multivariate linear regression model equation  $y = \epsilon + \theta_0 + \theta_c X_c$ :

- $y$ : Predicted performance
- $\epsilon$ : Prediction error
- $\theta_0$ : Global bias
- $\theta_c$ : Weight of characteristic  $c$
- $X_c$ : Characteristic  $c$  for all samples

# GENERATE (LINEAR) EXPLANATIONS



We aim to fit the following multivariate linear regression model:

$$\mathbf{y} = \epsilon + \theta_0 + \theta_c \mathbf{X}_c.$$

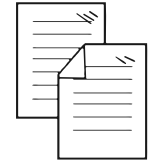
We use the ordinary least squares (OLS) optimization model:

$$(\theta_0^*, \theta_c^*) = \min_{\theta_0, \theta_c} \frac{1}{2} \|\mathbf{y} - \theta_0 - \theta_c \mathbf{X}_c\|_2^2.$$

# EXPLANATORY FRAMEWORK: STEPS







# 05

## RESULTS

# IMPACT OF CHARACTERISTICS

Characteristics' weights  
 $[\theta_0, \theta_1, \dots, \theta_c]$

Characteristics	LightGCN		DGCF		UltraGCN		SVD-GCN	
	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla
$R^2$ (adj. $R^2$ )	0.971(0.971)	0.979(0.978)	0.973(0.973)	0.982(0.981)	0.965(0.964)	0.860(0.858)	0.982(0.981)	0.981(0.981)
<i>Constant</i>	0.100***	0.121***	0.089***	0.107***	0.061***	0.062***	0.116***	0.135***
<i>SpaceSize<sub>log</sub></i>	0.070***	0.192***	0.133***	0.237***	-0.059***	0.318***	0.064***	0.114***
<i>Shape<sub>log</sub></i>	-0.253*	-0.231	-0.282**	-0.220*	0.135	-0.003	-0.193	-0.232*
<i>Density<sub>log</sub></i>	0.194***	0.298***	0.243***	0.327***	0.026*	0.321***	0.203***	0.234***
<i>Gini-U</i>	0.296**	0.104	0.074	-0.071	-0.043	-0.931***	0.136	0.143
<i>Gini-I</i>	1.362***	0.681***	1.108***	0.560***	0.605***	-0.144	1.138***	0.748***
<i>AvgDegree-U<sub>log</sub></i>	0.390***	0.605***	0.518***	0.673***	-0.100*	0.640***	0.364***	0.464***
<i>AvgDegree-I<sub>log</sub></i>	0.137*	0.374***	0.235***	0.453***	0.034	0.637***	0.171**	0.231**
<i>AvgClustC-U<sub>log</sub></i>	0.613***	0.665***	0.726***	0.783***	-0.077	0.706***	0.613***	0.496***
<i>AvgClustC-I<sub>log</sub></i>	0.087	0.332*	0.168	0.373**	0.062	0.671**	0.057	0.215
<i>Assort-U</i>	0.094***	0.024*	0.093***	0.013	0.123***	-0.019	0.080***	0.010
<i>Assort-I</i>	-0.051	-0.031	-0.056*	-0.055	0.001	-0.174***	-0.048*	-0.088*

\*\*\* $p$ -value  $\leq 0.001$ , \*\* $p$ -value  $\leq 0.01$ , \* $p$ -value  $\leq 0.05$

# IMPACT OF CHARACTERISTICS

How good is the linear regression at making predictions?

Characteristics	LightGCN		DGCF		UltraGCN		SVD-GCN	
	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla
$R^2(\text{adj. } R^2)$	0.971(0.971)	0.979(0.978)	0.973(0.973)	0.982(0.981)	0.965(0.964)	0.860(0.858)	0.982(0.981)	0.981(0.981)
<i>Constant</i>	0.160	0.121	0.089	0.107	0.001	0.002	0.110	0.135
<i>SpaceSize<sub>log</sub></i>	0.070***	0.192***	0.133***	0.237***	-0.059***	0.318***	0.064***	0.114***
<i>Shape<sub>log</sub></i>	-0.253*	-0.231	-0.282**	-0.220*	0.135	-0.003	-0.193	-0.232*
<i>Density<sub>log</sub></i>	0.194***	0.298***	0.243***	0.327***	0.026*	0.321***	0.203***	0.234***
<i>Gini-U</i>	0.296**	0.104	0.074	-0.071	-0.043	-0.931***	0.136	0.143
<i>Gini-I</i>	1.362***	0.681***	1.108***	0.560***	0.605***	-0.144	1.138***	0.748***
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<i>Assort-U</i>	0.094***	0.024*	0.093***	0.013	0.123***	-0.019	0.080***	0.010
<i>Assort-I</i>	-0.051	-0.031	-0.056*	-0.055	0.001	-0.174***	-0.048*	-0.088*

\*\*\* $p\text{-value} \leq 0.001$ , \*\* $p\text{-value} \leq 0.01$ , \* $p\text{-value} \leq 0.05$

$R^2$  is, for many settings, above 95%.

The regression model can provide good explanations.

# IMPACT OF CHARACTERISTICS

Characteristics	LightGCN		DGCF		UltraGCN		SVD-GCN	
	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla
$R^2$ (adj. $R^2$ )	0.971(0.971)	0.979(0.978)	0.973(0.973)	0.982(0.981)	0.965(0.964)	0.860(0.858)	0.982(0.981)	0.981(0.981)
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\*\*\* $p$ -value  $\leq 0.001$ , \*\* $p$ -value  $\leq 0.01$ , \* $p$ -value  $\leq 0.05$

# IMPACT OF CHARACTERISTICS

Characteristics	LightGCN		DGCF		UltraGCN		SVD-GCN	
	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla
$R^2$ (adj. $R^2$ )	0.971(0.971)	0.979(0.978)	0.973(0.973)	0.982(0.981)	0.965(0.964)	0.860(0.858)	0.982(0.981)	0.981(0.981)
Constant	0.100***	0.121***	0.089***	0.107***	0.061***	0.062***	0.116***	0.135***
SpaceSize <sub>log</sub>	0.070***	0.192***	0.133***	0.237***	-0.059***	0.318***	0.064***	0.114***
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\*\*\*p-value  $\leq 0.001$ , \*\*p-value  $\leq 0.01$ , \*p-value  $\leq 0.05$

inverse

direct

Graph CF = neighborhood + latent factors.

Graph CF behaves as factorization-based models.

# IMPACT OF CHARACTERISTICS

Characteristics	LightGCN		DGCF		UltraGCN		SVD-GCN	
	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla
$R^2$ (adj. $R^2$ )	0.971(0.971)	0.979(0.978)	0.973(0.973)	0.982(0.981)	0.965(0.964)	0.860(0.858)	0.982(0.981)	0.981(0.981)
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<i>Assort-I</i>	-0.051	-0.031	-0.056*	-0.055	0.001	-0.174***	-0.048*	-0.088*

topological

\*\*\* $p$ -value  $\leq 0.001$ , \*\* $p$ -value  $\leq 0.01$ , \* $p$ -value  $\leq 0.05$

# IMPACT OF CHARACTERISTICS

Characteristics	LightGCN		DGCF		UltraGCN		SVD-GCN	
	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla
$R^2$ (adj. $R^2$ )	0.971(0.971)	0.979(0.978)	0.973(0.973)	0.982(0.981)	0.965(0.964)	0.860(0.858)	0.982(0.981)	0.981(0.981)
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<i>Assort-I</i>	-0.051	-0.031	-0.056*	-0.055	0.001	-0.174***	-0.048*	-0.088*

direct



\*\*\* $p$ -value  $\leq 0.001$ , \*\* $p$ -value  $\leq 0.01$ , \* $p$ -value  $\leq 0.05$

This **confirms** what observed in the **models'** formulations.

**High users' interactions** correspond to better performance.

# IMPACT OF CHARACTERISTICS

Characteristics	LightGCN		DGCF		UltraGCN		SVD-GCN	
	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla
$R^2$ (adj. $R^2$ )	0.971(0.971)	0.979(0.978)	0.973(0.973)	0.982(0.981)	0.965(0.964)	0.860(0.858)	0.982(0.981)	0.981(0.981)
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<i>Assort-I</i>	-0.051	-0.031	-0.056*	-0.055	0.001	-0.174***	-0.048*	-0.088*

strong  
direct



\*\*\* $p$ -value  $\leq 0.001$ , \*\* $p$ -value  $\leq 0.01$ , \* $p$ -value  $\leq 0.05$

Values for **users**- and **items**-side are **more flattened** for **UltraGCN** and **SVD-GCN**.



# IMPACT OF CHARACTERISTICS

Characteristics	LightGCN		DGCF		UltraGCN		SVD-GCN	
	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla
$R^2$ (adj. $R^2$ )	0.971(0.971)	0.979(0.978)	0.973(0.973)	0.982(0.981)	0.965(0.964)	0.860(0.858)	0.982(0.981)	0.981(0.981)
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\*\*\* $p$ -value  $\leq 0.001$ , \*\* $p$ -value  $\leq 0.01$ , \* $p$ -value  $\leq 0.05$

direct



inverse



LightGCN and DGCF have larger coefficients than SVD-GCN, because they explicitly propagate messages.

# IMPACT OF CHARACTERISTICS

Characteristics	LightGCN		DGCF		UltraGCN		SVD-GCN	
	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla	Yelp2018	Gowalla
$R^2$ (adj. $R^2$ )	0.971(0.971)	0.979(0.978)	0.973(0.973)	0.982(0.981)	0.965(0.964)	0.860(0.858)	0.982(0.981)	0.981(0.981)
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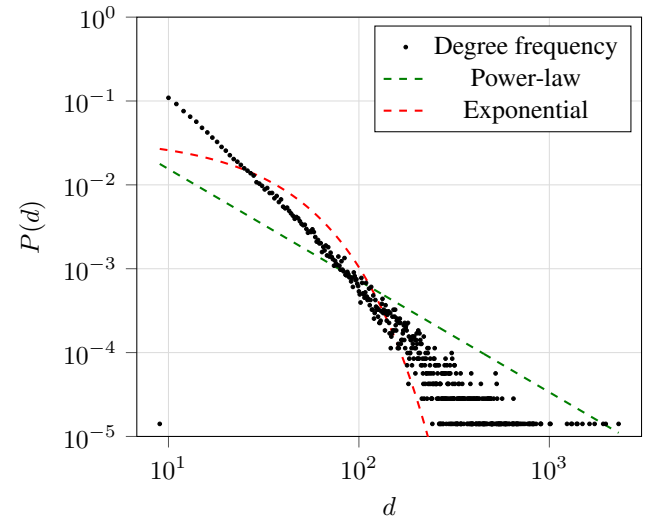
\*\*\* $p$ -value  $\leq 0.001$ , \*\* $p$ -value  $\leq 0.01$ , \* $p$ -value  $\leq 0.05$

direct

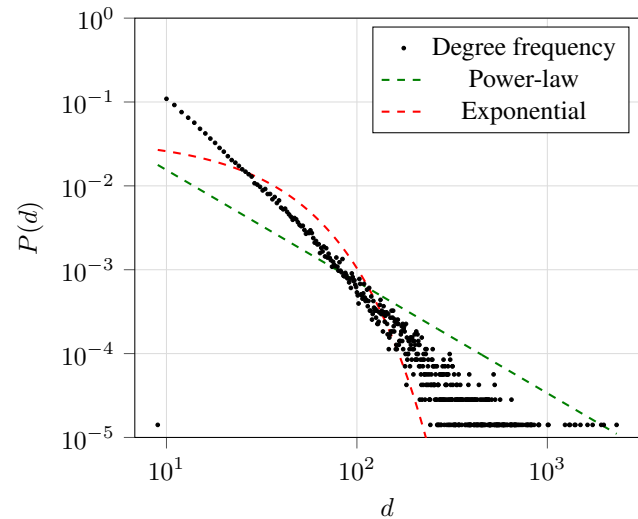
inverse

UltraGCN has bigger coefficients, probably because it uses the infinite-layer message passing.

# Probability distribution of node degrees on Gowalla



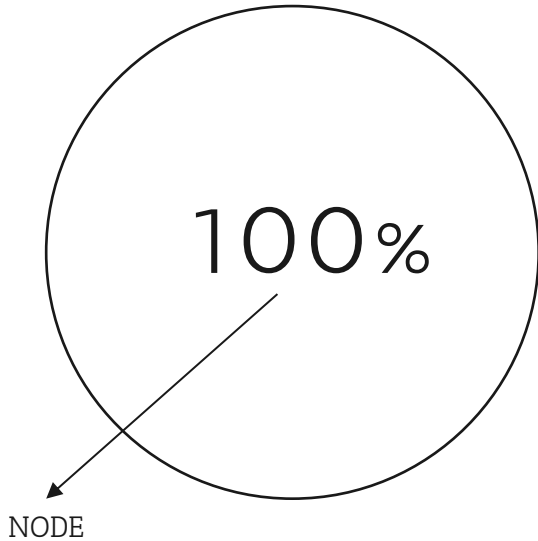
In the **worst-case** scenario, **node-dropout** drops many **high-degree** nodes; **edge-dropout**, drops all the **edges connected to several nodes** and thus **disconnect them** from the graph.



Is this **undermining** the **goodness** of the  
proposed **explanatory framework**?

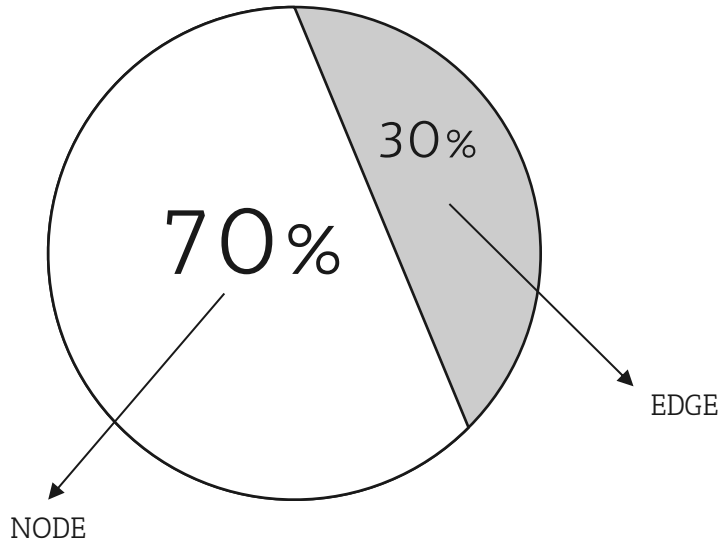


# INFLUENCE OF NODE- AND EDGE-DROPOUT



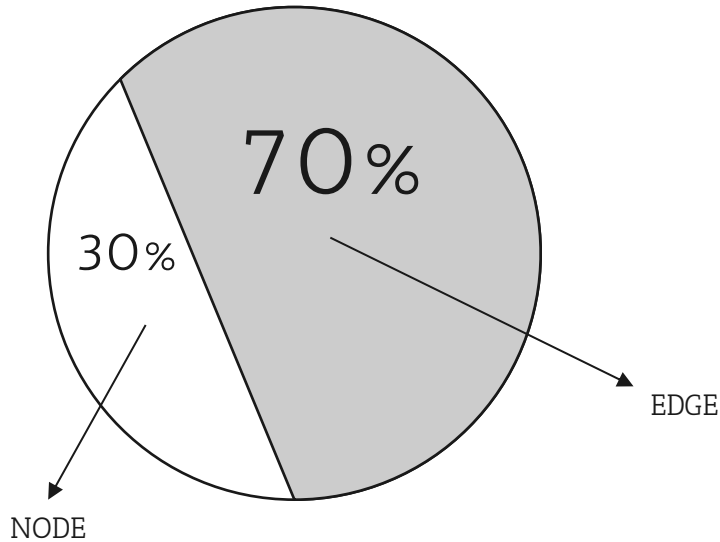
[Setting A] 100% node-dropout

# INFLUENCE OF NODE- AND EDGE-DROPOUT



[Setting B] 70% node-droput  
30% edge-droput

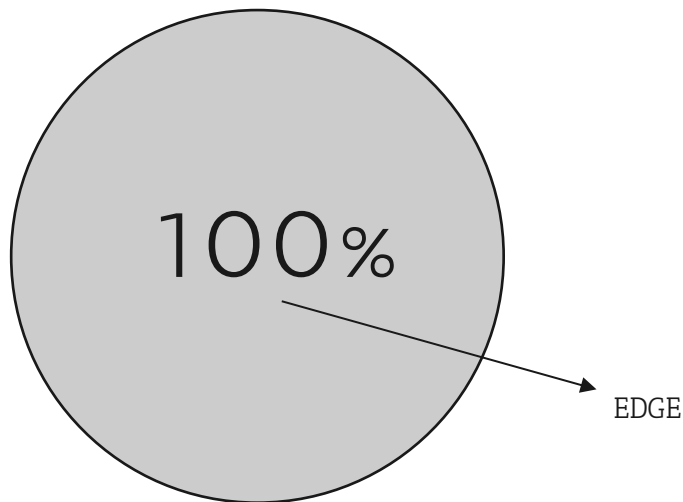
# INFLUENCE OF NODE- AND EDGE-DROPOUT



[Setting C] 30% node-droput  
70% edge-droput



# INFLUENCE OF NODE- AND EDGE-DROPOUT



[Setting D] 100% edge-dropout

# INFLUENCE OF NODE- AND EDGE-DROPOUT

Datasets' statistics

Characteristics	Node drop ●●● Edge drop ○○○		Node drop ●●○ Edge drop ●○○		Node drop ●○○ Edge drop ●●○		Node drop ○○○ Edge drop ●●●	
	LightGCN	SVD-GCN	LightGCN	SVD-GCN	LightGCN	SVD-GCN	LightGCN	SVD-GCN
<i>R</i> <sup>2</sup> (adj. <i>R</i> <sup>2</sup> )	0.597(0.583)	0.754(0.745)	0.968(0.967)	0.970(0.969)	0.986(0.985)	0.987(0.987)	0.994(0.994)	0.991(0.991)
<i>Constant</i>	0.179***	0.193***	0.146***	0.159***	0.098***	0.112***	0.062***	0.077***
<i>SpaceSize<sub>log</sub></i>	0.092	0.037	0.143***	0.064**	0.220***	0.136***	-0.162***	-0.048
<i>Shape<sub>log</sub></i>	-0.078	-0.118	-0.265	-0.261*	-0.175	-0.303	0.084	-0.157
<i>Density<sub>log</sub></i>	-0.079**	-0.090**	0.261***	0.195***	0.323***	0.251***	0.072**	0.040
<i>Gini-U</i>	-0.013	0.015	0.184	0.193	0.246	0.224	0.291***	0.225*
<i>Gini-I</i>	0.883***	0.884***	0.856***	0.867***	0.911***	0.940***	0.389***	0.387***
<i>AvgDegree-U<sub>log</sub></i>	0.052	0.005	0.536***	0.390***	0.631***	0.539***	-0.132*	0.070
<i>AvgDegree-I<sub>log</sub></i>	-0.026	-0.112	0.271**	0.129	0.456***	0.236*	-0.048	-0.087
<i>AvgClustC-U<sub>log</sub></i>	0.209	0.168	0.654***	0.508**	0.687**	0.647***	-0.133	0.016
<i>AvgClustC-I<sub>log</sub></i>	-0.141	-0.227	0.137	-0.007	0.436	0.172	-0.145	-0.112
<i>Assort-U</i>	0.008	-0.001	0.017	0.008	0.013	-0.002	0.011	-0.003
<i>Assort-I</i>	-0.022	-0.078**	0.028	-0.057	0.059	-0.056	0.012	-0.037

\*\*\*p-value ≤ 0.001, \*\*p-value ≤ 0.01, \*p-value ≤ 0.05

Node-dropout retains smaller portions of the dataset than edge-dropout.

# INFLUENCE OF NODE- AND EDGE-DROPOUT

Characteristics	Node drop ●●● Edge drop ○○○		Node drop ●●○ Edge drop ●○○		Node drop ●○○ Edge drop ●●○		Node drop ○○○ Edge drop ●●●	
	Average Sampling Statistics		Average Sampling Statistics		Average Sampling Statistics		Average Sampling Statistics	
	Users: 5,828	Items: 7,887	Users: 12,744	Items: 17,229	Users: 21,730	Items: 29,316	Users: 28,526	Items: 38,467
	Interactions: 45,620		Interactions: 97,785		Interactions: 160,919		Interactions: 209,659	
	LightGCN	SVD-GCN	LightGCN	SVD-GCN	LightGCN	SVD-GCN	LightGCN	SVD-GCN
$R^2$ (adj. $R^2$ )	0.597(0.583)	0.754(0.745)	0.968(0.967)	0.970(0.969)	0.986(0.985)	0.987(0.987)	0.994(0.994)	0.991(0.991)
Constant	0.179***	0.193***	0.146***	0.159***	0.098***	0.112***	0.062***	0.077***
SpaceSize <sub>log</sub>	0.092	0.037	0.143***	0.064**	0.220***	0.136***	-0.162***	-0.048
Shape <sub>log</sub>	-0.078	-0.118	-0.265	-0.261*	-0.175	-0.303	0.084	-0.157
Density <sub>log</sub>	-0.079**	-0.090**	0.261***	0.195***	0.323***	0.251***	0.072**	0.040
Gini-U	-0.013	0.015	0.184	0.193	0.246	0.224	0.291***	0.225*
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AvgClustC-I <sub>log</sub>	-0.141	-0.227	0.137	-0.007	0.436	0.172	-0.145	-0.112
Assort-U	0.008	-0.001	0.017	0.008	0.013	-0.002	0.011	-0.003
Assort-I	-0.022	-0.078**	0.028	-0.057	0.059	-0.056	0.012	-0.037

\*\*\*p-value  $\leq 0.001$ , \*\*p-value  $\leq 0.01$ , \*p-value  $\leq 0.05$

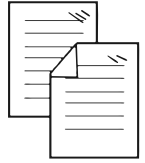
Meaningful learned dependencies (this setting is like ours).

# INFLUENCE OF NODE- AND EDGE-DROPOUT

Characteristics	Node drop ●●● Edge drop ○○○		Node drop ●●○ Edge drop ●○○		Node drop ●○○ Edge drop ●●○		Node drop ○○○ Edge drop ●●●	
	Average Sampling Statistics		Average Sampling Statistics		Average Sampling Statistics		Average Sampling Statistics	
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Assort-I	-0.022	-0.078**	0.028	-0.057	0.059	-0.056	0.012	-0.037

\*\*\*p-value  $\leq 0.001$ , \*\*p-value  $\leq 0.01$ , \*p-value  $\leq 0.05$

Not-statistically-significant at the extremes.



06

**TAKE-HOME  
MESSAGES**

# WHAT WE HAVE LEARNED

## 01

LATENT-FACTORS *vs.*  
NEIGHBORHOOD

# WHAT WE HAVE LEARNED

01

LATENT-FACTORS *vs.*  
NEIGHBORHOOD

02

NODE DEGREE

# WHAT WE HAVE LEARNED

01

LATENT-FACTORS *vs.*  
NEIGHBORHOOD

02

NODE DEGREE

03

CLUSTERING  
COEFFICIENT



# WHAT WE HAVE LEARNED

01

LATENT-FACTORS vs.  
NEIGHBORHOOD

02

NODE DEGREE

03

CLUSTERING  
COEFFICIENT

04

DEGREE ASSORTATIVITY

# WHAT WE HAVE LEARNED

01

LATENT-FACTORS *vs.*  
NEIGHBORHOOD

02

NODE DEGREE

03

CLUSTERING  
COEFFICIENT

04

DEGREE ASSORTATIVITY

05

NODE- *vs.* EDGE-  
DROPOUT

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# THANKS!

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